

# HARVARD UNIVERSITY

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## HARVARD LOGIC COLLOQUIUM

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### *Virtual Large Cardinal Principles*

DATE: Wednesday, November 8

TIME: 4:00–5:00 pm

LOCATION: Logic Center, Room 420, 2 Arrow Street

Given a set-theoretic property  $\mathcal{P}$  characterized by the existence of elementary embeddings between some first-order structures, let's say that  $\mathcal{P}$  holds *virtually* if the embeddings between structures from  $V$  characterizing  $\mathcal{P}$  exist somewhere in the generic multiverse. We showed with Schindler that virtual versions of supercompact,  $C^{(n)}$ -extendible,  $n$ -huge and rank-into-rank cardinals form a large cardinal hierarchy consistent with  $V = L$ . Included in the hierarchy are virtual versions of inconsistent large cardinal notions such as the existence of an elementary embedding  $j : V_\lambda \rightarrow V_\lambda$  for  $\lambda$  much larger than the supremum of the critical sequence. The Silver indiscernibles, under  $0^\sharp$ , which have a number of large cardinal properties in  $L$ , are also natural examples of virtual large cardinals. Virtual versions of forcing axioms, including PFA, SCFA, and resurrection axioms, have been studied by Schindler and Fuchs, who showed that they are equiconsistent with virtual large cardinals. We showed with Bagaria and Schindler that the virtual version of Vopěnka's Principle is consistent with  $V = L$ . Bagaria had showed that Vopěnka's Principle holds if and only if the universe has a proper class of  $C^{(n)}$ -extendible cardinals for every  $n \in \omega$ . We almost generalized his result by showing that the virtual version is equiconsistent with the existence, for every  $n \in \omega$ , of a proper class of virtually  $C^{(n)}$ -extendible cardinals. With Hamkins we showed that Bagaria's result cannot generalize by constructing a model of virtual Vopěnka's Principle in which there are no virtually extendible cardinals. The difference arises from the failure of Kunen's Inconsistency in the virtual setting. In the talk, I will discuss a mixture of results about the virtual large cardinal hierarchy and virtual Vopěnka's Principle.