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# HARVARD LOGIC COLLOQUIUM

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### Stronger tree properties and the SCH

DATE:	Thursday, March 8
TIME:	4:00–5:00 pm
LOCATION:	Logic Center, Room 420, 2 Arrow Street

Stronger tree properties capture the combinatorial essence of large cardinals. More precisely, for an inaccessible cardinal  $\kappa$ ,  $\kappa$  has the strong, resp. super, tree property if and only if  $\kappa$  is strongly compact, resp. supercompact. An old project in set theory is to get the tree property at every regular cardinal greater than  $\omega_1$ . Even more ambitiously, can we get stronger tree properties at all regular cardinals above  $\omega_1$ ? A positive answer would require many violations the singular cardinal hypothesis (SCH). This leads to the question whether the strong tree property implies SCH above. A positive answer would be an analogue of Solovay's theorem that SCH holds above a strongly compact cardinal.

We will show that consistently we can have the super tree property (ITP) at some  $\lambda$  together with failure of SCH above  $\lambda$ , for a non limit singular cardinal. The case of a limit singular cardinal is still open. We will also show that there is a model where ITP holds at the double successor of a singular and there are club many non internally unbounded models. This is another result in the direction of showing that ITP does not imply SCH above. Finally, we will discuss the situation for smaller cardinals like  $\aleph_{\omega+2}$ .

This is joint work with Sherwood Hachtman, University of Illinois at Chicago