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HARVARD LOGIC COLLOQUIUM

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When is time forward isomorphic to time backward?

DATE: Wednesday, November 13
TIME: 4:30–5:30 pm
LOCATION: Logic Center, Room 420, 2 Arrow Street

When is forward time is isomorphic to backward time for a given dynamical system? When the acting group is $\mathbb{Z}$, this asks when a transformation T is (measure theoretically) isomorphic to its inverse. It was not until 1951, that Anzai refuted a conjecture of Halmos and von Neumann by exhibiting the first example of a transformation where T is not isomorphic to its inverse.

In this talk, I show that there is a a one-to-one computable method of associating to each Π^0_1 formula in number theory, a computable measure preserving diffeomorphism T_ϕ of the two-torus so that

- ϕ is true

if and only if

- T_ϕ is measure theoretically isomorphic to T_ϕ^{-1}.

The class of problems is large enough to include the *Riemann Hypothesis*, *Goldbach's Conjecture* and statements such as "*Zermelo-Frankel Set Theory (ZFC) is consistent*." In consequence, each of these formulas is *equivalent* to whether $T \cong T^{-1}$ for a diffeomorphism of $\mathbb{T}^2$ canonically associated to that formula.

Thus there is an ergodic diffeomorphism of the two-torus T_{RH} such that $T_{RH} \cong T_{RH}^{-1}$ if and only if the Riemann Hypothesis holds, and a different ergodic diffeomorphism T_{GC} such that $T_{GC} \cong T_{GC}^{-1}$ if and only if Goldbach's conjecture holds and so forth. Moreover there is a canonical diffeomorphism T such that the question $T \cong T^{-1}$ is independent of ZFC.